Accurate description of non-stationary random seismic excitation

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Abstract. Seismic random processes are characterized by high non-stationarity and, in most cases, by a marked variability of frequency contents. The widely used hypothesis modelling seismic signal as a simple product of stationary signal and a deterministic modulation function, consequently, is hardly always applicable. Such assumption leads to an incorrect estimation of the frequency contents, which can significantly influence the assessment of structural response to such a seismic event. As a solution, the multicomponent decomposition of the non-stationary seismic record is presented. The wavelet multiresolution analysis is used as a tool. An example of the seismic response of a simplified structure is given.

1. Introduction

The present stage of computer development allows us to perform the simulation of effects of the recorded seismic events on structures. However, every earthquake is a unique event and for the regular statistical processing we miss the adequate set of realizations. Hence we try to determine realistic characteristics of a particular seismic event, which are able to describe it (in the stochastic manner) with sufficient accuracy. Using these characteristics it should be possible to calculate statistical properties of the response of the structure.

The usual form, which is used for representation of non-stationary seismic processes, is a product of the deterministic modulation and a stationary signal

$$v(t)= m(t)v_0(t) \qquad (1)$$

However, such a simple formula can be applied to the simplest cases only, as it supposes identical frequency structure throughout the seismic event. There is no support for such assumptions. On the contrary, various types of seismic waves or a dispersion of waves due to the inhomogeneity of the subsoil definitely cause significant variation of the frequency contents of seismic records. The use of such a simple form has also dangerous consequences: Fourier transform, the tool for the stationary case only, should not be used, as well as the power spectral density (PSD), which is defined for stationary random processes only. There is no guarantee, that the numerical PSD estimator gives a reasonable result in this case.

It is evident, that it is necessary to construct and use a model, which takes such variability of the frequency contents into account. This can be done using the evolutionary power spectral density [1].

2. Decomposition of seismic excitation

2.1. Simple decomposition

Vibrations of the system, caused by a seismic event, serve as a typical example of random kinematic excitation produced by the motion of supports. Whatever is the character of the structure, the function $v(t)$, describing the movement of foundation soil on the support site, is considered as a known random continuous Gaussian non-stationary process. These processes are usually described in the form (1), where $v_0(t)$ is supposed to be Gaussian stationary process and $m(t)$ is the deterministic modulation function. However, such multiplicative modulation cannot assure anything more than the roughly constant mean amplitude in time.

An ideal modulation function should be able to describe a sudden and strong start and a slow decay of the earthquake as well as the case when the strongest impulse is delayed or the signal contains several beats. Among the most used functions belongs double exponential function (2), or the well-known function (3) proposed by Saragoni and Hart [2]

$$m(t) = ah(t)\left(e^{-\alpha t} - e^{-\beta t}\right), \qquad m(t) = ah(t)t^{\beta}e^{-\alpha t} \qquad (2,3)$$

where $h(t)$ stands for the Heaviside unit step function. Both mentioned functions satisfy sufficiently first demand, but they fail in the case of complicated signal. They are usually sufficient for synthetic seismograms, but they are not able to follow complicated envelope of a true seismogram.

For such purpose we prefer more complicated but more flexible spline modulation function. Such a modulation function can be symbolically written in the form of

$$m(t) = \sum_{i=0}^{k} \alpha_i B^N (t - t_i) \qquad (4)$$

where $0=t_0<\ldots.<t_k=T$ is a partition of the time interval and B^N is the B-spline of degree N. The polynomial degree can be chosen arbitrarily, but the value $N=2$ or 3 is completely adequate.

2.2. Multicomponent decomposition

The basic idea here is to split the original process into a sum of processes with (narrow) band-limited PSD. In such a case the variation of the frequency contents with time can be considered as negligible. In the next step each one of the components can be described using the decomposition (1) and modulation (4). This procedure leads to an approximation of the evolutionary power spectra of the non-stationary process. Therefore, we write

$$v(t) = \sum_{i=0}^{p} v_i = \sum_{i=0}^{p} m_i(t) v_{i0}(t) \qquad (5)$$

A feasible basis for resolution (5) is provided by wavelet multiresolution analysis applied [3]. Such a technique does not impose any requirements on a priori stationarity and periodicity of the process.

2.3. Spectral density approximation

The stationary part of the excitation, either $v_0(t)$ or $v_{i0}(t)$, is usually assumed to be an AR or ARMA process. The well-known Kanai-Tajimi spectra (6) or the one proposed by Bolotin (7)

$$G(\omega) = \frac{G_0 c^2 \left(c^2 + 4b^2\omega^2\right)}{\left(c^2 - \omega^2\right)^2 + 4b^2\omega^2 c^2}, \qquad \psi(\omega) = \frac{\sigma_0^2 a^2 b}{\pi\left(a^2 - \omega^2\right)^2 + 4b^2\omega^2} \qquad (6,7)$$

are the spectral densities of the continuous ARMA(2,1) and AR(2) processes respectively. The form of continuous AR(2) model provides the possibility of construction of a finite differential filter yielding the process $v_{i0}(t)$, generated as an output on the basis of the Gaussian white noise.

The difference between both AR(2) and ARMA(2,1) does not seem to be crucial. Although the more complicated ARMA(2,1) form fits usually the computed PSD approximation better than AR(2), the difference between both approximation is usually smaller than uncertainty of the PSD estimation.

3. Response of a structure

The movement of a linear discrete or discretized structure under kinematic excitation in supports can be described by the system

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{B}\dot{\mathbf{u}}(t) + \mathbf{C}\mathbf{u}(t) = \mathbf{F}\dot{\mathbf{v}}(t) + \mathbf{G}\mathbf{v}(t) \qquad (8)$$

where **M**, **B**, **C, F**, **G** are matrices of parameters of the system; $\mathbf{u}(t)$ denotes the system response in free nodes; and $\mathbf{v}(t)$ is the vector of kinematic excitation by random seismic processes in supports.

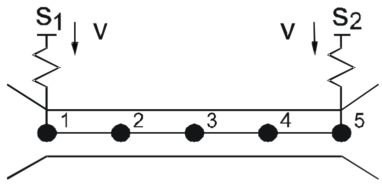

FIG. 1. Lumped mass modelled bridge

Statistical properties of the response of the structure described by eq. (8) can be obtained using various methods: There exists analytical formulas for computing the dispersion of the response of a linear structure, known as integral spectral analysis method (see [4]) and generalized correlation method [5]. Both of them are suitable for the excitation in the form of multicomponent decomposition, with PSD in the form of continuous AR(2) model and spline modulation function. As a representative of numerical methods we mention the stochastic Newmark method, introduced by To in [6]. It can adopt an arbitrarily modulated discrete ARMA(p,q) random process as excitation.

4. Seismic response of a simple structure

Let us demonstrate the whole procedure on a simplified structure, see Fig. 1. As an example of a seismic excitation we have chosen Sierra Madre earthquake recorded at Altadena, Eaton Canyon Park station, June 28, 1991, E-W component, epicentral distance 49km, peak accel. 1.756 m/s^2. The record was corrected and double integrated to obtain displacement history.

The Fig. 2 shows 3 narrow-band components obtained using the wavelet decomposition in its first column. The second column gives the respective modulation (quadratic spline). Third column shows the corresponding stationary random processes. Their spectral densities are depicted in the last column. The solid curve corresponds to the estimated true PSD, the dashed line follows the AR(2) approximation (7) and the dotted line shows the ARMA(2,1) approximation (6).

The stochastic response of the bridge subjected to the seismic excitation in both supports $S_{1,2}$ was computed using the generalized correlation method and the stochastic Newmark method. The resulting variances of the individual nodes are indicated in the Fig. 3. The Fig. 3a shows results of the analysis with the input data obtained using the decomposition (1), while the Fig. 3b demonstrates result of the generalized correlation method and utilizing multicomponent decomposition according to Fig. 2.

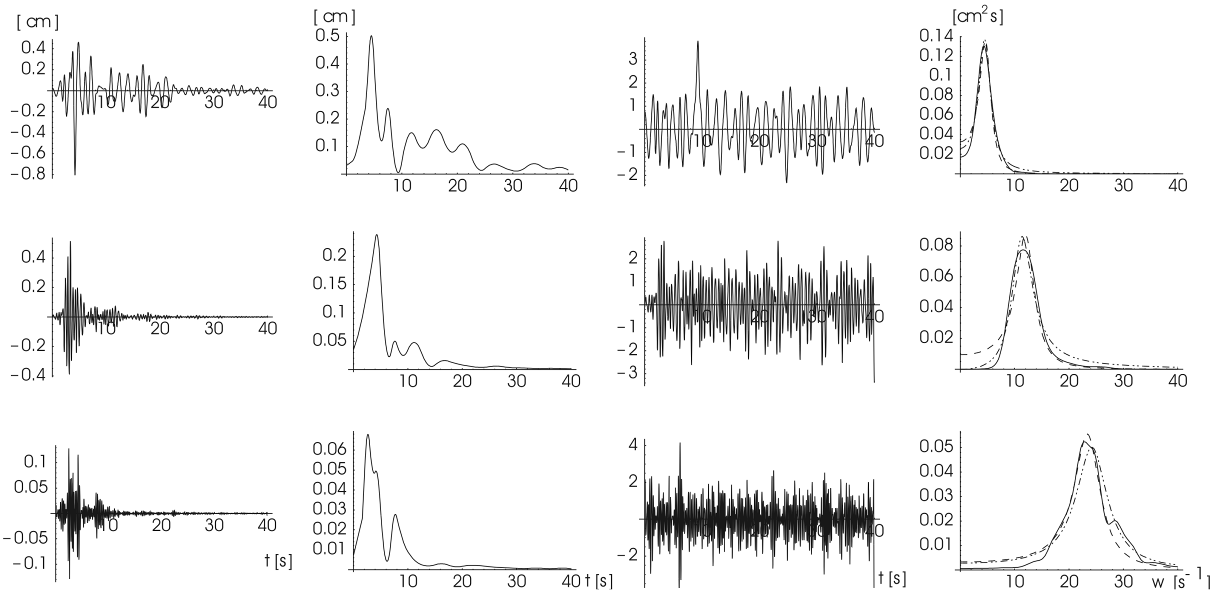

FIG. 2. Sample analysis of three components of the wavelet decomposition of the earthquake record

At the first sight one might be surprised by finding out that the maximum value of the dispersion of the response (b) is smaller than half of the response in the case (a). The explanation of this fact is given by detailed considering of the Fig. 3c with the course of the approximated spectral densities of the first three details of dispersion together with the approximated PSD derived according to simple decomposition (5). The value of the PSD of the first detail for $\omega=6.4s^{-1}$, (i.e. the first eigenfrequency of the bridge), is almost one half ($0.043cm^2s$) of the value of the PSD computed from the stationarized part of the classically decomposed seismogram ($0.076\ cm^2s$). The incorrect estimate of PSD of $v_0(t)$ (see (1)) was caused by its poor stationarity.

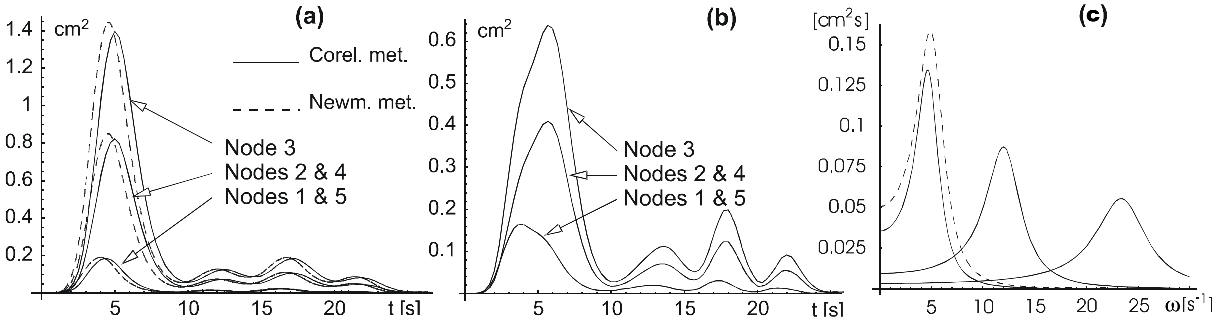

FIG. 3. (a) Displacement variance of the structure computed using simple decomposition
(b) Variance computed using multicomponent decomposition of the Sierra Madre record
(c) Spectral densities used in computation: cases (a) – dashed line and (b) – solid lines

5. Conclusions

It is coming to light, that the main source of problems in determining of the response of the structure to (random) excitation is not solely the calculation, but rather the determination of correct characteristics of the excitation process. The "stationary" process, obtained using the simple decomposition (1), usually lacks the true stationarity, which prevents us from estimating its PSD correctly. This problem can be solved using the multicomponent decomposition. Moreover, the multicomponent decomposition allows us to describe much broader spectra using a set of simple AR(2) processes. It is worth to mention that it is necessary to use the B-spline modulation function while deriving the stationary part of a non-stationary signal.

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